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This tripartite analysis of a logarithm is more powerful than may appear at a first reading. We learn by contrast; and the value of this particular explanation lies in seeing logarithms from the viewpoints of the computer, the printer, and the graph. Most teachers are inclined to believe that the best instruction consists in presenting to the student only one idea at a time. Psychologically, however, it is equally sound to keep together two closely related ideas, and teach by contrast and comparisons. Ordinarily logarithms are taught solely for their use in computing; later in analytic geometry the student draws perhaps a few exponential curves; and then in calculus he learns about the base e. The introduction of both bases simultaneously impresses the significant facts about each; the mind is put in a receptive mood when it sees the necessity of the rules. The writer believes the present paper affords a good illustration of how various parts hitherto separated in the curriculum by years can be taught sequentially when organized on the basis of functions. In addition to unity of subject matter, we also obtain an unusual degree of coherence. Each day's work is intimately linked with the preceding and prepares for the following.

NOTE ON LAGRANGE'S LIKE-PRODUCING QUADRINOMIAL.[1]

By Sir THOMAS MUIR, South Africa.

1. From an identity originally due to Lagrange we know that the product

$$(x_1^2 + bcx_2^2 + cax_3^2 + abx_4^2)(y_1^2 + bcy_2^2 + cay_3^2 + aby_4^2)$$

can be expressed in the same form as either factor, namely, in the form

$$P_1^2 + bcP_2^2 + caP_3^2 + abP_4^2.$$

For the case, however, where the second factor is the same as the first, the formula is nugatory, all that it then tells us being that $(x_1^2 + bcx_2^2 + cax_3^2 + abx_4^2)^2$ is equal to itself. It seems therefore not a little interesting to know that without introducing any new principle we can show that the product of the *cube* of the first factor by the second factor can be simply expressed in the particular form in question.

2. We recall in the first place that

$$(x_1^2 + bcx_2^2 + cax_3^2 + abx_4^2)^2 = \begin{vmatrix} x_1 & bcx_2 & cax_3 & abx_4 \\ -x_2 & x_1 & -ax_4 & ax_3 \\ -x_3 & bx_4 & x_1 & -bx_2 \\ -x_4 & -cx_3 & cx_2 & x_1 \end{vmatrix}$$

and that each primary minor of this determinant contains the square root of the determinant as a factor, the adjugate of the determinant being

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[1] Part of the object of this paper is to throw a little light on the subject of Problem 489 of this MONTHLY for Nov. 1917.

$$\begin{vmatrix} x_1Q & x_2Q & x_3Q & x_4Q \\ -bcx_2Q & x_1Q & -bx_4Q & cx_3Q \\ -cax_3Q & ax_4Q & x_1Q & -cx_2Q \\ -abx_4Q & -ax_3Q & bx_2Q & x_1Q \end{vmatrix}$$

if we put Q for $x_1^2 + bcx_2^2 + cax_3^2 + abx_4^2$.

3. From this it follows that if we take any three columns of the said determinant along with *any fourth column whatever* to form a new determinant, the latter must have Q for a factor. A special interest attaches to the choosing of y_1, y_2, y_3, y_4 for the fourth column: for the new determinants and their factors then are

$$\begin{vmatrix} y_1 & bcx_2 & cax_3 & abx_4 \\ y_2 & x_1 & -ax_4 & ax_3 \\ y_3 & bx_4 & x_1 & -bx_2 \\ y_4 & -cx_3 & cx_2 & x_1 \end{vmatrix} = Q(x_1y_1 - bcx_2y_2 - cax_3y_3 - abx_4y_4),$$

$$\begin{vmatrix} y_1 & x_1 & cax_3 & abx_4 \\ y_2 & -x_2 & -ax_4 & ax_3 \\ y_3 & -x_3 & x_1 & -bx_2 \\ y_4 & -x_4 & cx_2 & x_1 \end{vmatrix} = Q(-x_2y_1 - x_1y_2 - ax_4y_3 + ax_3y_4),$$

$$\begin{vmatrix} y_1 & x_1 & bcx_2 & abx_4 \\ y_2 & -x_2 & x_1 & ax_3 \\ y_3 & -x_3 & bx_4 & -bx^2 \\ y_4 & -x_4 & -cx_3 & x^1 \end{vmatrix} = Q(x_3y_1 - bx_4y_2 + x_1y_3 + bx_2y_4),$$

$$\begin{vmatrix} y_1 & x_1 & bcx_2 & cax_3 \\ y_2 & -x_2 & x_1 & -ax_4 \\ y_3 & -x_3 & bx_4 & x_1 \\ y_4 & -x_4 & -cx_3 & cx_2 \end{vmatrix} = Q(-x_4y_1 - cx_3y_2 + cx_2y_3 - x_1y_4),$$

where the cofactors of Q turn out to be values of P_1, P_2, P_3, P_4 in § 1 as given by Lagrange's rule. The consequence is that if in the equality of § 1 we substitute for each P the quotient of the appropriate determinant by Q, we obtain the following theorem:

If by $|\,rstu\,|$ *be denoted the determinant whose columns are the* rth, sth, tth, uth *columns of the array*

$$\begin{array}{ccccc} y_1 & x_1 & bcx_2 & cax_3 & abx_4 \\ y_2 & -x_2 & x_1 & -ax_4 & ax_3 \\ y_3 & -x_3 & bx_4 & x_1 & -bx_2 \\ y_4 & -x_4 & -cx_3 & cx_2 & x_1 \end{array}$$

then

$$(x_1^2 + bcx_2^2 + cax_3^2 + abx_4^2)^3 \cdot (y_1^2 + bcy_2^2 + cay_3^2 + aby_4^2)$$

$$= |\,1345\,|^2 + bc\,|\,1245\,|^2 + ca\,|\,1235\,|^2 + ab\,|\,1234\,|^2.$$

4. From Lagrange's result it of course follows that

$$(x_1^2 + bcx_2^2 + cax_3^2 + abx_4^2)(y_1^2 + bcy_2^2 + \cdots)(z_1^2 + bcz_2^2 + \cdots)$$

can be expressed in the same form as any one of the factors, say in the form

$$T_1^2 + bcT_2^2 + caT_3^2 + abT_4^2,$$

and there thus arises the problem of finding the T's in terms of the x's, y's, z's and a, b, c. The simplest way of writing the four expressions obtained is by means of the notation for bipartite functions, according to which

$$\begin{array}{ccc|c} x & y & z & \\ \hline a & b & c & \xi \\ d & e & f & \eta \\ g & h & i & \zeta \end{array} \quad \text{stands for } ax\xi + by\xi + cz\xi + dx\eta + ey\eta + fz\eta + gx\zeta + hy\zeta + iz\zeta,$$

each element of the square array being taken along with the outside element in the same column and at the same time along with the outside element in the same row. We then have

$$T_1 = \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline y_1 & bcy_2 & cay_4 & aby_4 & z_1 \\ -y_2 & y_1 & -ay_4 & ay_3 & bcz_2 \\ -y_3 & by_4 & y_1 & -by_2 & caz_3 \\ -y_4 & -cy_3 & cy_2 & y_1 & abz_4 \end{array}$$

and the other T's differing from it merely in having for their outside column a different row of the determinant equivalent for $(z_1^2 + bcz_2^2 + caz_3^2 + abz_4^2)^2$.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence.

DISCUSSIONS.

We believe that those of our readers who have followed the discussions of certain solutions of cubic equations as published in the February and June numbers of the MONTHLY will be especially interested in the method given below for approximating nearly equal roots of a cubic.